Renormalization of Black Hole Entropy

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We review the renormalization of one-loop effective action for gravity coupled to a scalar field and that of the Bekenstein-Hawking entropy of a black hole plus the statistical entropy of the scalar field. It is found that the total entropy of the black hole's geometric entropy and the statistical entropy yields the renormalized Bekenstein-Hawking area-law of black hole entropy only for even dimensional Reissner-Nördstrom (Schwarzschild) black holes. We discuss the problem of the microscopic origin of black hole entropy in connection with the renormalization of black hole entropy.

I. INTRODUCTION

One of the mysterious and challenging phenomena in general relativity is black hole physics. Black holes formed from the gravitational collapse of massive stars are characterized by mass M, angular momentum J and charge Q, regardless of the process of their formation. Classically a black hole prevents any particle from crossing its event horizon from the interior, and all the particles falling into it eventually run into the singularity. Thus black holes are classically black. On the other hand, quantum mechanically the black hole emits a thermal spectrum characterized by the Hawking temperature and has the Bekenstein-Hawking entropy [1,2].

The Bekenstein-Hawking entropy of black hole, also known as the area-law, is given by one quarter of the area of the event horizon of the black hole. The black hole entropy is a quantity entirely determined by the geometry of black hole. However, in statistical mechanics the entropy of a system is determined by the number of microscopically indistinguishable states available to the system. Thus the Bekenstein-Hawking entropy is lacking in the explanation of the microscopic origin of entropy. As an attempt to understand the microscopic origin of black hole entropy t' Hooft introduced a brick wall model, in which a scalar field is confined to a spherical shell enclosing the event horizon and when one chooses a thickness having the geometric meaning of an invariant distance from the event horizon, the statistical entropy of the scalar field computed in terms of the thickness of the brick wall gives the correct black hole entropy [3]. This idea was more elaborated by considering the renormalization of the total entropy of the black hole's geometric entropy and the statistical entropy of the scalar field in the black hole background [4]. The renormalization scheme of black hole entropy was realized for Reissner-Nördstrom (RN) black hole minimally coupled to a massive scalar field in four [5] and six dimensions [6].

In this talk we revisit the problem of the renormalization of black hole entropy for black holes coupled to a massive scalar field. By extending the previous result [6,7], we show explicitly that the renormalization of the black hole's geometric entropy plus the statistical entropy of the scalar field leads correctly to the renormalized Bekenstein-Hawking black hole entropy only for all the even dimensional RN black holes.

The organization of this talk is as follows. In Sec. II we review the Bekenstein-Hawking black hole entropy for a four-dimensional RN black hole and the problem of the microscopic origin of black hole entropy. In Sec. III we derive the Hawking temperature for the D-dimensional RN black holes. In Sec. IV we apply the 't Hooft brick wall model to D-dimensional RN black holes to explain the black hole entropy in terms of the statistical entropy. In Sec. V we review the renormalization of the one-loop effective action for gravity minimally coupled to a massive scalar field for the even dimensional RN black holes. Finally in Sec. VI we show explicitly the renormalization of the black hole entropy plus the statistical entropy for all the even dimensional RN black holes. We discuss the possible role of supersymmetry in the black hole entropy problem.

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II. BEKENSTEIN-HAWKING BLACK HOLE ENTROPY AND STATISTICAL ENTROPY

It was discovered by Bekenstein and Hawking that a black hole has a complete analog with a thermal system with the Hawking temperature and the Bekenstein-Hawking entropy. A four-dimensional RN black hole has the temperature and entropy

$$T_4 = \frac{1}{\pi r_+}\Big(1 - \frac{r_-}{r_+}\Big), \qquad S_4 = \frac{A_4}{4G}, \tag{1}$$

where $A_4 = 4\pi r_+^2$ is the area of the event horizon of the RN black hole with the metric

$$\begin{aligned} ds^2 = &-\Big(1 - \frac{r_-}{r}\Big)\Big(1 - \frac{r_+}{r}\Big)dt^2 \\ &+ \frac{dr^2}{\Big(1 - \frac{r_-}{r}\Big)\Big(1 - \frac{r_+}{r}\Big)} + r^2 d\Omega_2^2, \end{aligned} \tag{2}$$

where r_+ and r_- are the outer and inner event horizons, respectively. Use of geometrodynamical units $c = \hbar = k_B = 1$ is made, but G will be kept whenever necessary.

We follow the beautiful derivation by Gibbons and Hawking [8] of the geometric entropy of the four-dimensional RN black hole using the Euclidean path integral. The partition function for gravity plus matter typically represented by ϕ is given by

$$\mathcal{Z}_4 = e^{-\beta F_4} = \int \mathcal{D}[g]\mathcal{D}[\phi]e^{-\mathcal{I}_E[g,\phi]} \tag{3}$$

where $\mathcal{I}_E$ is the Wick rotation of the Einstein-Hilbert action

$$\mathcal{I}_{EH} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}R + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d\Sigma[K], \tag{4}$$

where $K = g^{ab}K_{ab}$, K_{ab} being the second fundamental form on $\partial\mathcal{M}$, and $[K] = K_{|\partial\mathcal{M}} - K_{|\mathrm{flatspace}}$. For the RN black hole, the bulk contribution vanishes due to the on-shell condition (being a solution of the Einstein equation), and only the surface contribution survives to yield

$$\mathcal{I}_{EH} = \frac{1}{8\pi G}\frac{\partial}{\partial n}\int_{\partial\mathcal{M}} d\Sigma. \tag{5}$$

For the RN black hole the Einstein-Hilbert action is evaluated

$$\mathcal{I}_{EH} = i\frac{\pi r_+^2}{G}. \tag{6}$$

¿From the partition function one finds the Helmholtz free energy

$$F_4 = \frac{\mathcal{I}_E}{\beta} = -\frac{\pi r_+^2}{G\beta}, \tag{7}$$

and the entropy

$$S_4 = \beta^2\frac{\partial F_4}{\partial\beta} = \frac{A_4}{4G}. \tag{8}$$

Eq. (8) is the famous Bekenstein-Hawking black hole entropy. Note that the Bekenstein-Hawking entropy is determined entirely by the geometry of black hole itself.

On the other hand, in statistical mechanics the entropy of a system is defined by the number of microscopically indistinguishable states N:

$$S = \ln(N). \tag{9}$$

For a system in a thermal equilibrium the entropy is also given by

$$S = -\mathrm{Tr}\Big[\hat{\rho}\ln(\hat{\rho})\Big] \tag{10}$$

where $\hat{\rho}$ is the density operator

$$\hat{\rho} = \frac{1}{Z}e^{-\beta\hat{H}}. \tag{11}$$

By evaluating the partition function

$$\begin{aligned} Z &= \mathrm{Tr}\Big[e^{-\beta\hat{H}}\Big] \\ &\equiv e^{-\beta F}, \end{aligned} \tag{12}$$

one finds the entropy as before

$$S = \beta^2\frac{\partial F}{\partial\beta}. \tag{13}$$

Thus one may raise a question about the microscopic origin of black hole entropy. The area-law of Bekenstein-Hawking black hole entropy is rooted entirely on the geometry of black hole, whereas the entropy in statistical mechanics counts the number of microscopically indistinguishable states available to the system. Then, where does the entropy of black hole reside? There have recently been several different approaches toward this problem of the microscopic origin of black hole entropy. Among these string theory seems to explain the correct number of string states $e^{\frac{A}{4G}}$ [9]. However, one of the problems with which string theory confronts now is to explain the topology of black holes from the spacetime of strings.

III. ENTROPY OF D-DIMENSIONAL RN BLACK HOLES

The D-dimensional RN black hole has the metric [10]

$$ds^2 = -\Delta(r)dt^2 + \Delta^{-1}(r)dr^2 + r^2d\Omega^2_{D-2}, \tag{14}$$

where

$$\Delta(r) = \left(1-\left(\frac{r_-}{r}\right)^{D-3}\right)\left(1-\left(\frac{r_+}{r}\right)^{D-3}\right), \tag{15}$$

where r_+ and r_- are the outer and inner event horizons given by

$$r_\pm = \left[\frac{4\Gamma\Big((D-1)\frac{1}{2}\Big)}{(D-2)\pi^{(D-3)/2}}\Big(M \pm \sqrt{M^2-Q^2}\Big)\right]^{\frac{1}{D-3}}. \tag{16}$$

For the nonextremal black holes we can find the coordinate transformation that removes the apparent singularity of the event horizon. With a transformation of the form

$$r = r_+ + c\rho^q, \tag{17}$$

the two dimensional metric near the event horizon becomes

$$-\Delta(r)dt^2 + \frac{dr^2}{\Delta(r)} \simeq -\frac{(D-3)c(1-u)}{r_+}dt^2 + \frac{cq^2r_+}{(D-3)(1-u)}\rho^{q-2}d\rho^2, \tag{18}$$

where $u = \left(\frac{r_-}{r_+}\right)^{D-3}$. By choosing the undetermined parameters

$$q = 2, \quad c = \frac{(D-3)(1-u)}{4r_+}, \tag{19}$$

and by introducing a new coordinate

$$\theta = \frac{(D-3)(1-u)}{2r_+}t, \tag{20}$$

we transform the black hole metric up to the order of $\mathcal{O}(\rho^2)$

$$\begin{aligned} ds^2 = & -\rho^2\left[1 + \frac{(D-3)}{4r+^2}\Big((D-3)u - \frac{(D-2)(1-u)}{2}\Big)\rho^2\right]d\theta^2 \\ & + \left[1 - \frac{(D-3)}{4r+^2}\Big((D-3)u - \frac{(D-2)(1-u)}{2}\Big)\rho^2\right]d\rho^2 + r_+^2\left[1 + \frac{(D-3)(1-u)}{4r_+^2}\rho^2\right]d\Omega_{D-2}^2. \end{aligned} \tag{21}$$

We further transform the Lorentzian metric into a Euclidean metric by a Wick rotation

$$\theta = i\theta_E. \tag{22}$$

With the periodicity imposed

$$\theta_E = \frac{(D-3)(1-u)}{2r_+}\tau_E = 2\pi, \tag{23}$$

the Euclidean metric near the event horizon becomes topologically $R^2 \otimes S_{D-2}$:

$$\begin{aligned} ds_E^2 = & \rho^2\left[1 + \frac{(D-3)}{4r+^2}\Big((D-3)u - \frac{(D-2)(1-u)}{2}\Big)\rho^2\right]d\theta_E^2 \\ & + \left[1 - \frac{(D-3)}{4r+^2}\Big((D-3)u - \frac{(D-2)(1-u)}{2}\Big)\rho^2\right]d\rho^2 + r_+^2\left[1 + \frac{(D-3)(1-u)}{4r_+^2}\rho^2\right]d\Omega_{D-2}^2. \end{aligned} \tag{24}$$

This periodicity defines the Hawking temperature of the RN black hole.

$$\beta_H = \frac{1}{\tau_E} = \frac{4\pi r_+}{(D-3)(1-u)}. \tag{25}$$

Note that the temperature can also be obtained from

$$\beta_H = \frac{4\pi}{\Delta'(r_+)}. \tag{26}$$

As in Sec. II, by evaluating the partition function, one finds the Bekenstein-Hawking entropy

$$S_D = \frac{A_D}{4G}, \tag{27}$$

where

$$A_D = \frac{2\pi^{\frac{D}{2}}}{\Gamma\Big(\frac{D}{2}\Big)}r_+^{D-1} \tag{28}$$

is the area of the event horizon.

IV. BRICK WALL MODEL

As an attempt to understand the origin of black hole entropy, 't Hooft considered the statistical entropy of a scalar field outside the event horizon of a Schwarzschild black hole [3]. In order to regulate the ultraviolet divergence of the Helmholtz free energy due to the infinite degeneracy of the number of states on the event horizon, he introduced a brick wall of an infinitesimal thickness. In this Section we apply his idea to the massive scalar field in the D-dimensional RN black hole.

The massive scalar field in the RN black hole background obeys the Klein-Gordon equation

$$(-g)^{-1/2}\partial_\mu\Big(g^{\mu\nu}(-g)^{1/2}\partial_\nu\Phi\Big)-m^2\Phi=0. \tag{29}$$

We find a stationary solution in the spherical coordinates

$$\Phi(x,t)=e^{-iEt}\phi(r,x_i), \tag{30}$$

where $x_i, i=2,3,\cdots,D$, are the coordinates on a $(D-2)$-sphere whose metric is related with the metric g_S on the unit-sphere:

$$g^{ij}=\frac{1}{r^2}g_S^{ij}. \tag{31}$$

Then the radial motion becomes

$$\begin{aligned}\frac{1}{r^{D-2}}\frac{\partial}{\partial r}\Big(r^{D-2}\Delta(r)\frac{\partial}{\partial r}\phi\Big)-&\\ \Big(\frac{l(l+D-3)}{r^2}+m^2\Big)\phi+\frac{E^2}{\Delta(r)}\phi=0.&\end{aligned} \tag{32}$$

Eq. (32) is an analog of one-dimensional Schrödinger equation. We find the radial momentum $p_r=\frac{\partial\mathcal{S}}{\partial r}$ using the WKB wave function $\phi\propto e^{i\mathcal{S}(r,l,E)}$:

$$p_r(r,l,E)=\frac{1}{\Delta^{1/2}(r)}\left[\frac{E^2}{\Delta(r)}-\frac{l(l+D-3)}{r^2}-m^2\right]^{1/2}. \tag{33}$$

The key idea of the 't Hooft brick wall is to introduce the ultraviolet and infrared cut-offs to regulate the infinite quantities just according to the regularization method of quantum field theory. The scalar field is confined to a spherical shell of the inner radius $r_+ + h$ and the outer radius L and required to satisfy the Dirichlet boundary condition

$$\Phi(r=r_+ + h)=0=\Phi(r=L). \tag{34}$$

The brick wall thickness h cuts off the ultraviolet divergence and the outer radius L cuts off the infrared divergence. We are interested in the physics inside the spherical shell. The number of radial modes is given by

$$n(l,E)=\frac{1}{\pi}\int_{r_+ + h}^{L}dr p_r(r,l,E), \tag{35}$$

and, by summing over the angular momentum states with the correct degeneracy of the angular momentum states taken into account, we obtain the total number of states for a given energy E

$$g(E)=\frac{1}{\pi}\int_{r_+ + h}^{L}\frac{dr}{\Delta(r)}\int dl(2l+D-3)\frac{(l+D-4)!}{(D-3)!(l!)}\left[E^2-\Big(m^2+\frac{l(l+D-3)}{r^2}\Big)\right]^{1/2}. \tag{36}$$

The most divergent terms as h tends to zero are found [6]

$$g_{2n}^{\rm m.div}(E)=\frac{B(n-1,\frac{3}{2})}{\pi(n-1)(2n-3)(2n-3)!}r_+^{2n-1}\left(\frac{(1-u)^2m^2+2uE^2}{(1-u)^2m^2}\right)^{n-1/2}\frac{E^{2n-1}}{\epsilon^{n-1}}, \tag{37}$$

for an even-dimensional case of $D=2n$, and

$$g_{2n+1}^{\rm m.div}(E)=\frac{1}{2^{2n-1}(n-\frac{1}{2})(2n-2)(n-1)!n!}r_+^{2n}\left(\frac{(1-u)^2m^2+2uE^2}{(1-u)^2m^2}\right)^{n}\frac{E^{2n-1}}{\epsilon^{n-1/2}}, \tag{38}$$

for an odd-dimensional case of $D=2n+1$, where $\epsilon=(D-3)\frac{h}{r_+}$ and $u=\left(\frac{r_-}{r_+}\right)^{D-3}$. The Helmholtz free energy is given by

$$F = -\frac{1}{\pi}\int_0^\infty dE \frac{g(E)}{e^{\beta E}-1}. \tag{39}$$

By making use of the integral

$$\int_0^\infty dx \frac{x^{n-1}}{e^x - 1} = \Gamma(n)\zeta(n), \tag{40}$$

where ζ is the Rieman zeta function, in the large mass limit we find the free energy in $2n$ dimensions

$$F_{2n}^{\mathrm{m.div}} = -\frac{B(n-1,\frac{3}{2})}{\pi^2(n-1)(2n-3)(2n-3)!}\frac{\Gamma(2n)\zeta(2n)}{\beta^{2n}}\frac{r_+^{2n-1}}{\epsilon^{n-1}}, \tag{41}$$

and in $2n+1$ dimensions

$$F_{2n+1}^{\mathrm{m.div}} = -\frac{1}{\pi 2^{2n-1}(n-\frac{1}{2})(2n-2)(n-1)!n!}\frac{\Gamma(2n)\zeta(2n)}{\beta^{2n}}\frac{r_+^{2n}}{\epsilon^{n-1/2}}. \tag{42}$$

Finally we obtain the on-shell entropy of the scalar field

$$S_{2n}^{\mathrm{m.div}} = -\frac{2(2n)(2n-1)\zeta(2n)B(n-1,\frac{3}{2})}{\pi^2(2n-3)}\Big(\frac{(2n-3)(1-u)}{4\pi}\Big)^{2n-1}\frac{1}{\epsilon^{n-1}}, \tag{43}$$

in $2n$ dimensions, and

$$S_{2n+1}^{\mathrm{m.div}} = -\frac{(2n)!\zeta(2n)}{\pi 2^{2n-1}(n-\frac{1}{2})(2n-2)(n-1)!n!}\Big(\frac{(2n-2)(1-u)}{4\pi}\Big)^{2n-1}\frac{r_+}{\epsilon^{n-1/2}}, \tag{44}$$

in $2n+1$ dimensions, respectively. By suitably choosing the brick wall thickness ϵ, the statistical entropy of the scalar field can give rise to the Bekenstein-Hawking entropy

$$S_D = \frac{A_D}{4}. \tag{45}$$

Despite of an ad hoc prescription of the brick wall, this procedure for obtaining the black hole entropy may have a physical origin, since the thickness has the geometric meaning of an invariant distance from the event horizon.

V. RENORMALIZED ONE-LOOP EFFECTIVE ACTION

From now on we shall focus on the $2n$-dimensional RN black hole, since the renormalization scheme was shown to work only in four [5] and six dimensions, but not in five dimensions [6], and, as will be shown in the next Section, it works for all the even-dimensional RN black holes. So let us consider the one-loop effective action for $2n$-dimensional gravity coupled to a massive scalar field [11]

$$\mathcal{I}_{2n} = \int d^{2n}x\sqrt{-g}\Big[-\frac{\Lambda}{8\pi G} + \frac{R}{16\pi G} + \frac{\alpha_1}{4\pi}R^2 + \frac{\alpha_2}{4\pi}R_{\mu\nu}R^{\mu\nu} + \frac{\alpha_3}{4\pi}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\Big] \tag{46}$$

where Λ_B is the bare cosmological constant, G_B the bare gravitational constant, and $\alpha_{B,i}$ are the bare coupling constants. Using the DeWitt-Schwinger method we find the one-loop effective action for the massive scalar field [11,12]

$$W_{2n}(m) = \frac{1}{2\big(4\pi\big)^n}\int d^{2n}x\sqrt{-g}\int_0^\infty d(is)\sum_{k=0}^\infty a_k(x)\big(is\big)^{k-n-1}e^{-im^2 s}, \tag{47}$$

where

$$\begin{aligned} a_0 &= 1, \quad a_1 = \frac{1}{6}R, \\ a_2 &= \frac{1}{30}R_{,\mu}{}^{;\mu} + \frac{1}{72}R^2 \\ &\quad + \frac{1}{180}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - \frac{1}{180}R_{\mu\nu}R^{\mu\nu}. \end{aligned} \tag{48}$$

In order to regulate the divergent terms of the effective action W_{2n} we shall use the Pauli-Villars regularization method, in which we introduce a number of fictitious bosonic and fermionic regulator fields of masses m_{B_i} and m_{F_i}, respectively. Then the total action is the sum of those of regulator and scalar fields

$$W_{2n} = \frac{1}{2\left(4\pi\right)^n} \int d^{2n}x\sqrt{-g} \int_0^\infty d(is) \sum_{k=0}^{\infty} a_k(x)\left(is\right)^{k-n-1}\big(\sum_i e^{-im_{B_i}^2 s} - \sum_i e^{-im_{F_i}^2 s}\big). \tag{49}$$

Performing the integral one finds the divergent contributions to the effective action [6]

$$\begin{aligned} W_{2n}^{\rm div} = & \frac{1}{2\left(4\pi\right)^n} \int d^{2n}x\sqrt{-g} \sum_{k=0}^{n} a_k(x)(-1)^{n+1-k} \\ & \times\Bigg[\frac{1}{(n-k)!}\Big(\sum_i m_{B_i}^{2(n-k)} \ln\big(m_{B_i}^2\big) - \sum_i m_{F_i}^{2(n-k)} \ln\big(m_{F_i}^2\big)\Big) \\ & -\frac{1}{(n-k)!}\Big(I_1 + \sum_{p=1}^{n-k} \frac{1}{p}\Big)\Big(\sum_i m_{B_i}^{2(n-k)} - \sum_i m_{F_i}^{2(n-k)}\Big) \\ & + \sum_{l=2}^{n+1-k} \frac{(-1)^l}{(n+1-k-l)!} I_l\Big(\sum_i m_{B_i}^{2(n+1-k-l)} - \sum_i m_{F_i}^{2(n+1-k-l)}\Big)\Bigg], \end{aligned} \tag{50}$$

where

$$I_p = \int_0^\infty \frac{1}{t^p}. \tag{51}$$

To remove the infinite constants I_p given by Eq. (51), we impose the mass conditions

$$\sum_i m_{B_i}^{2(n-k)} = \sum_i m_{F_i}^{2(n-k)} \tag{52}$$

for $k = 0, 1, \cdots, n$. We are then left with the renormalized action

$$W_{2n}^{\rm ren} = \int d^{2n}x\sqrt{-g} \sum_{k=0}^{n} a_k(x) \frac{\mathcal{B}_k}{2\left(4\pi\right)^n (n-k)!}, \tag{53}$$

where

$$\begin{aligned} \mathcal{B}_k = (-1)^{n+1-k} & \Big(\sum_i m_{B_i}^{2(n-k)} \ln\big(m_{B_i}^2\big) \\ & - \sum_i m_{F_i}^{2(n-k)} \ln\big(m_{F_i}^2\big)\Big) \end{aligned} \tag{54}$$

are the renormalization constants. We may now renormalize the one-loop effective action for gravity plus matter field by redefining the cosmological, gravitational, and coupling constants

$$\begin{aligned} & \frac{\Lambda}{8\pi G} - \frac{\mathcal{B}_0}{2(4\pi)^n n!} = \frac{\Lambda^{\rm ren}}{8\pi G^{\rm ren}}, \\ & \frac{1}{16\pi G} + \frac{\mathcal{B}_1}{12(4\pi)^n (n-1)!} = \frac{1}{16\pi G^{\rm ren}}, \\ & \frac{\alpha_1}{4\pi} + \frac{\mathcal{B}_2}{144(4\pi)^n (n-2)!} = \frac{\alpha_1^{\rm ren}}{4\pi}, \\ & \frac{\alpha_2}{4\pi} - \frac{\mathcal{B}_2}{360(4\pi)^n (n-2)!} = \frac{\alpha_2^{\rm ren}}{4\pi}, \\ & \frac{\alpha_3}{4\pi} + \frac{\mathcal{B}_2}{360(4\pi)^n (n-2)!} = \frac{\alpha_3^{\rm ren}}{4\pi}. \end{aligned} \tag{55}$$

VI. RENORMALIZATION OF RN BLACK HOLES IN $2N$-DIMENSIONS

We shall now compute the statistical entropy of the massive scalar field in the $2n$-dimensional RN black hole background. The statistical entropy of the scalar field in the RN black hole background has already been found in four [5] and six dimensions [6]. By applying the WKB idea to the quantum mechanical Klein-Gordon equation we are able to find the number of states, $g(E)$, for a given energy and to obtain the Helmholtz free energy (39).

By taking the degeneracy of angular momentum states for a fixed l, the number of states of the massive scalar field is given by

$$g_{2n}(E,m)=\frac{1}{\pi}\int_{r_++h}^{L}\frac{dr}{\Delta(r)}\int dl(2l+2n-3)\frac{(l+2n-4)!}{(2n-3)!l!}\sqrt{E^2-\Big(m^2+\frac{l(l+2n-3)}{r^2}\Big)\Delta(r)}, \tag{56}$$

where L is an infrared cutoff and h is a brick wall introduced to regulate the possible ultra-violet divergence. Following Ref. [6] we obtain the number of states

$$g_{2n}(E,m)=\frac{1}{\pi(2n-3)!}\sum_{k=0}^{n-2}C_k^{2n}B\Big(k+1,\frac{3}{2}\Big)\frac{r_+^{2k+3}}{2n-3}\times\int_\epsilon dx\frac{\Big[E^2-x(1-u+ux)m^2\Big]^{k+\frac{3}{2}}}{(1-x)^{1+\frac{2k+3}{2n-3}}x^{k+2}(1-u+ux)^{k+2}}, \tag{57}$$

where

$$\begin{aligned} x &= 1-\Big(\frac{r_+}{r}\Big)^{2n-3},\\ \epsilon &= (2n-3)\frac{h}{r_+},\\ u &= \Big(\frac{r_-}{r_+}\Big)^{2n-3}. \end{aligned} \tag{58}$$

As the explicit calculation in four and six dimensions shows, there are two typical types of terms: the terms to be removed by the same mass conditions (52) and the terms contributing to the free energy and entropy, which are later on to be renormalized by redefining the coupling constants (55). We are, however, interested in the contribution to the entropy proportional to the area of the event horizon after removing all the possible divergent terms which depend on the infinitely large masses of regulator fields.

After some gymnastic of algebra, we get the number of states [6]

$$\begin{aligned} g_{2n}(E,m) &= \frac{1}{\pi(2n-3)!}\sum_{k=0}^{n-2}C_k^{2n}B\Big(k+1,\frac{3}{2}\Big)\frac{r_+^{2k+3}}{2n-3}\sum_{q=0}^{\infty}H_q^{2n,k}\sum_{p=0}^{\infty}(-1)^p\binom{k+\frac{3}{2}}{p}\\ &\times\Big(\frac{(1-u)^2m^2+2uE^2}{(1-u)^2m^2}\Big)^{k+\frac{3}{2}}\Big(\frac{u}{(1-u)^2m^2+2uE^2}\Big)^p\sum_{l=0}^{p}\binom{p}{l}E^{4l}\int_\epsilon dx x^{-k-2+q}Z^{k+p-2l+\frac{3}{2}}, \end{aligned} \tag{59}$$

where $H_q^{2n,k}$ are the coefficients of Taylor expansion

$$\frac{1}{(1-x)^{1+\frac{2k+3}{2n-3}}(1-u+ux)^{k+2}}=\sum_{q=0}^{\infty}H_q^{2n,k}x^q, \tag{60}$$

and

$$Z=E^2-(1-u)m^2x. \tag{61}$$

A careful scrutiny shows that the term with $k=n-2, q=0, p=0, l=0$

$$g_{2n}^{\rm area}(E,m)=\frac{C_{n-2}^{2n}}{\pi\Gamma(2n-2)}B\Big(n-1,\frac{3}{2}\Big)\frac{r_+^{2n-1}}{2n-3}H_0^{2n,n-2}\int_\epsilon dx\frac{Z^{n-\frac{1}{2}}}{x^n} \tag{62}$$

contributes to the entropy proportional to the area of event horizon. It is found that

$$H_0^{2n,n-2} = \frac{1}{(1-u)^n}, \; C_{n-2}^{2n} = 1. \tag{63}$$

We now compute the integral

$$\begin{aligned} A_{n-\frac{1}{2}}^n &= \int_\epsilon dx \frac{Z^{n-\frac{1}{2}}}{x^n} \\ &= Z^{n+\frac{1}{2}}(\epsilon)\left[\frac{1}{(n-1)E^2}\frac{1}{\epsilon^{n-1}} + \sum_{l=1}^{n-2}(-1)^l \frac{1\cdot 3\cdot 5\cdots(2l+1)}{2^l(n-1)(n-2)\cdots(n-l-1)}\frac{1}{\epsilon^{n-l-1}}\frac{1}{E^2}\Big(\frac{(1-u)m^2}{E^2}\Big)^l\right] \\ &\quad + (-1)^n\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n-1}(n-1)!}\Big(\frac{(1-u)m^2}{E^2}\Big)^{n-1}\left[-2\sum_{l=1}^{n}\frac{E^{2n-2l}}{2l-1}Z^{l-\frac{1}{2}}(\epsilon) - E^{2n-1}\ln\Big(\frac{E-Z^{\frac{1}{2}}(\epsilon)}{E+Z^{\frac{1}{2}}(\epsilon)}\Big)\right]. \end{aligned} \tag{64}$$

Note that the term proportional to $m^{2n-2}E$ is

$$A_{n-\frac{1}{2}}^n = (1)^{n+1}\frac{(2n-1)!!}{2^{n-1}(n-1)!}\Big[(1-u)m^2\Big]^{n-1}E\ln\Big(\frac{(1-u)m^2}{4E^2}\epsilon\Big) \tag{65}$$

and that

$$B(n-1,\frac{3}{2}) = \frac{\pi^{1/2}\Gamma(n-1)}{2(n-\frac{1}{2})\Gamma(n-\frac{1}{2})}. \tag{66}$$

So the number of states becomes

$$g_{2n}^{\rm area}(E,m) = \frac{(-1)^{n+1}}{2^{2n-1}\pi^{1/2}(n-1)!\Gamma(n-\frac{1}{2})}\frac{r_+^{2n-1}}{(2n-3)(1-u)}m^{2n-2}E\ln\Big(\frac{(1-u)m^2}{4E^2}\epsilon\Big). \tag{67}$$

Finally we obtain the off-shell free energy

$$F_{2n}(m) = \frac{(-1)^{n+1}\pi^{3/2}}{122^{2n-2}\pi^{1/2}(n-1)!\Gamma(n-\frac{1}{2})}\frac{r_+^{2n-1}}{(2n-3)(1-u)\beta^2}m^{2n-2}\ln(m^2\epsilon), \tag{68}$$

and entropy

$$S_{2n}(m) = \frac{(-1)^{n}\pi^{3/2}}{122^{2n-3}\pi^{1/2}(n-1)!\Gamma(n-\frac{1}{2})}\frac{r_+^{2n-1}}{(2n-3)(1-u)\beta}m^{2n-2}\ln(m^2\epsilon). \tag{69}$$

Substituting the Hawking temperature (25) and the area of event horizon (28), we find the on-shell entropy of the scalar field

$$S_{2n}(m) = \frac{A_{2n}}{4}\frac{(-1)^n}{12(4\pi)^{n-1}(n-1)!}m^{2n-2}\ln(m^2\epsilon). \tag{70}$$

We introduce the same number of fictitious bosonic or fermionic regulator fields which obey the same Bose-Einstein distribution but with opposite signs [5]. To remove ultraviolet divergence of entropy, we use the Pauli-Villars regularization method. Let the masses of bosonic and fermionic regulator fields be m_{B_i} and m_{F_i}, respectively. Thus the total entropy is the sum of each field

$$S_{2n} = \sum_i S_{2n}(m_{B_i}) - \sum_i S_{2n}(m_{F_i}). \tag{71}$$

The divergence coming from the brick wall thickness is removed from the mass condition in Sec. V. In the end we obtain the renormalized entropy of the scalar field

$$S_{2n} = \frac{A_{2n}}{4}\frac{\mathcal{B}_1}{12(4\pi)^{n-1}}. \tag{72}$$

Thus we are able to show that the total entropy of the Bekenstein-Hawking entropy and the statistical entropy of the scalar field can be renormalized and lead to the correct renormalized black hole area-law

$$S_{2n}^{\rm ren} = \frac{A_{2n}}{4G} + \frac{A_{2n}}{4}\frac{\mathcal{B}_1}{12(4\pi)^{n-1}} = \frac{A_{2n}}{4G^{\rm ren}}. \tag{73}$$

Equation (73) is the main result of this talk.

VII. DISCUSSION AND CONCLUSION

In this talk we revisited the renormalization of black hole entropy. It was found that only for the even dimensional Reissner-Nördstrom black holes the total entropy of the black hole's geometric entropy and the statistical entropy of a massive scalar field gives the correct Bekenstein-Hawking black hole entropy in terms of the renormalized gravitational constant, which is the same as that appeared in the one-loop effective action for gravity minimally coupled to the massive scalar field. The covariant Pauli-Villars regularization method used to regularize both the one-loop effective action and the statistical entropy of the scalar field makes use of the fictitious regulator fields that obey the same Bose-Einstein distribution but contribute to the Helmholtz free energy with opposite signs. Even though not shown in this talk, this suggests that a black hole coupled to both bosonic scalar fields and fermionic fields may have the regularized black hole entropy. This implies that supersymmetry may play a role in understanding the microscopic origin of black hole entropy.

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